**SUBIECTUL I** **(30 de puncte)**

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|---|---|---|
| **1.** | $\left\|3-2\sqrt{3}\right\|=2\sqrt{3}-3$<br>$\left\|2\sqrt{3}-4\right\|=4-2\sqrt{3}$<br>Finalizare $\left\|3-2\sqrt{3}\right\|+\left\|2\sqrt{3}-4\right\|=1$ | **2p**<br>**2p**<br>**1p** |
| **2.** | $x^2+6x+2\geq -7 \Leftrightarrow x^2+6x+9\geq 0 \Leftrightarrow$<br>$\Leftrightarrow (x+3)^2\geq 0$, adevărat oricare ar fi $x\in\mathbb{R}$<br>sau<br>$y_V=-\frac{\Delta}{4a}=-7$ valoare minimă<br>Rezultă: $f(x)\geq -7$, oricare ar fi $x\in\mathbb{R}$ | **2p**<br>**3p** |
| **3.** | $x=$ preţul inţial al produsului $\Rightarrow \frac{84}{100}\cdot x=210$<br>Finalizare: $x=250$ lei | **3p**<br>**2p** |
| **4.** | Condiţii de existenţă: $x+1\geq 0 \Leftrightarrow x\in[-1;\infty)$<br>$\sqrt{x+1}=5 \Leftrightarrow x=24\in[-1;\infty)$ | **2p**<br>**3p** |
| **5.** | $d: x-2y+3=0 \Rightarrow m_d=\frac{1}{2}$<br>$d' \parallel d \Rightarrow m_{d'}=\frac{1}{2}$<br>Finalizare $d': y+2=\frac{1}{2}(x-1)\Rightarrow x-2y-5=0$ | **1p**<br>**2p**<br>**2p** |
| **6.** | $\sin 120^0=\sin 60^0=\frac{\sqrt{3}}{2}$<br>$\cos 150^0=-\cos 30^0=-\frac{\sqrt{3}}{2}$<br>Finalizare: $\sin 120^0+\cos 150^0=0$ | **2p**<br>**2p**<br>**1p** |

**SUBIECTUL II** **(30 de puncte)**

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| **a.** | $\hat{1}^{-1}=\hat{1}$<br>$\hat{2}^{-1}=\hat{3}$<br>$\hat{3}^{-1}=\hat{2}$<br>$\hat{4}^{-1}=\hat{4}$ | **2p**<br>**1p**<br>**1p**<br>**1p** |
| **b.** | Se verifică pe rând dacă elementele lui $\mathbb{Z}_5$ sunt soluţii.<br>Rezultă : $S=\{\hat{0};\hat{2};\hat{3}\}$ | **3p**<br>**2p** |
| **c.** | $\Delta=\begin{vmatrix}\hat{1} & \hat{2}\\ \hat{2} & \hat{3}\end{vmatrix}=\hat{4}$ inversabil în $\mathbb{Z}_5$. Rezultă sistemul are soluţie unică<br>$\Delta_x=\begin{vmatrix}\hat{0} & \hat{2}\\ \hat{3} & \hat{3}\end{vmatrix}=\hat{4}\Rightarrow x=\hat{1}$ | **2p**<br>**1p**<br>**1p** |

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| | $\Delta_y = \begin{vmatrix} \hat{1} & \hat{0} \\ \hat{2} & \hat{3} \end{vmatrix} = \hat{3} \Rightarrow y = \hat{2}$<br>Finalizare : $S = \{(\hat{1};\hat{2})\}$ | 1p |
| d. | $U(\mathbb{Z}_6) = \{\hat{1};\hat{5}\}$<br>Rezultă $P = \hat{1}\cdot\hat{5} = \hat{5}$ | 3p<br>2p |
| e. | $\hat{2}x + \hat{3} = \hat{5} \Leftrightarrow \hat{2}x = \hat{2} \Rightarrow$<br>$\Rightarrow x \in \{\hat{1};\hat{4}\}$ | 2p<br>3p |
| c. | $\begin{cases} x + \hat{2}y = \hat{0} \\ \hat{2}x + \hat{3}y = \hat{3} \end{cases} \Leftrightarrow \begin{cases} x = \hat{4}y \\ \hat{2}x + \hat{3}y = \hat{3} \end{cases} \Leftrightarrow$<br>$\begin{cases} x = \hat{4}y \\ \hat{5}y = \hat{3} \end{cases} \Leftrightarrow \begin{cases} y = \hat{3} \\ x = \hat{0} \end{cases} \Rightarrow$<br>$\Rightarrow S = \{(\hat{0};\hat{3})\}$ | 2p<br>2p<br>1p |

**SUBIECTUL III** **(30 de puncte)**

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| a. | $A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \Rightarrow$<br>$\Rightarrow A^3 = \begin{pmatrix} 1 & 0 \\ 0 & 8 \end{pmatrix}$ | 3p<br>2p |
| b. | $I_2 = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \Rightarrow a = c = 1, b = 0 \Rightarrow$<br>$I_2 \in M$ | 3p<br>2p |
| c. | $X = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \in M, Y = \begin{pmatrix} x & y \\ y & z \end{pmatrix} \in M \Rightarrow$<br>$\Rightarrow X + Y = \begin{pmatrix} a+x & b+y \\ b+y & c+z \end{pmatrix}, a+x, b+y, c+z \in \mathbb{Z} \Rightarrow$<br>$X + Y \in M$ | 1p<br>3p<br>1p |
| d. | $X = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \in M, Y = \begin{pmatrix} x & y \\ y & z \end{pmatrix} \in M \Rightarrow X \cdot Y = \begin{pmatrix} ax+by & ay+bz \\ bx+cy & by+cz \end{pmatrix}, Y \cdot X = \begin{pmatrix} xa+yb & xb+yc \\ ya+zb & yb+zc \end{pmatrix}$<br>Rezultă: $XY - YX = \begin{pmatrix} 0 & ay+bz-xb-yc \\ bx+cy-ya-zb & 0 \end{pmatrix} \Rightarrow$<br>$\det(XY - YX) = (ay+bz-xb-yc)^2 \geq 0$ | 2p<br>1p<br>2p |
| e. | $X = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, a,b,c \in \mathbb{Z} \Rightarrow X^2 = \begin{pmatrix} a^2+b^2 & ab+bc \\ ab+bc & b^2+c^2 \end{pmatrix} \Rightarrow$<br>$\begin{cases} a^2 + b^2 = 1 \\ b(a+c) = 0, a,b,c \in \mathbb{Z} \\ b^2 + c^2 = 1 \end{cases}$ | 1p<br>1p |

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| | Dacă $b=0 \Rightarrow a^2=c^2=1$.Deci: $X \in \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$<br>Dacă $b \neq 0 \Rightarrow b^2=1, a=c=0 \Rightarrow X \in \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \right\}$ | **2p**<br>**1p** |
| **f.** | $A^n = \begin{pmatrix} 1 & 0 \\ 0 & 2^n \end{pmatrix} \Rightarrow B = \begin{pmatrix} \underbrace{1+1+...+1}_{\text{de 2003 ori}} & 0 \\ 0 & 2+2^2+2^3+...+2^{2013} \end{pmatrix}$<br>Rezultă $B = \begin{pmatrix} 2013 & 0 \\ 0 & 2^{2014}-2 \end{pmatrix}$ | **3p**<br>**2p** |